Lattice QCD for EIC physics

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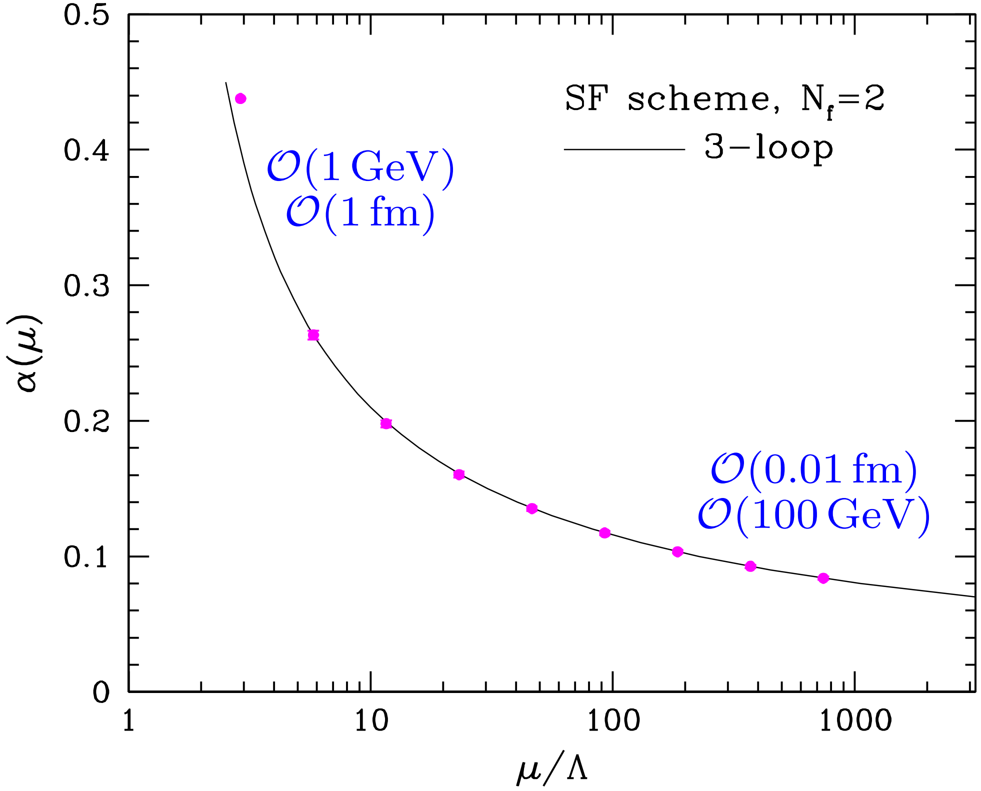
Outline

1. Introduction to lattice QCD:
   - the need for lattice
   - discretization procedure
   - simulations
   - systematic effects
2. Hadron structure from lattice QCD:
   - how it can be addressed on the lattice
   - moments of PDFs/GPDs
     $\rightarrow$ nucleon spin decomposition
   - x-dependence of PDFs/GPDs
3. Summary and prospects

ALPHA Collaboration, 2004

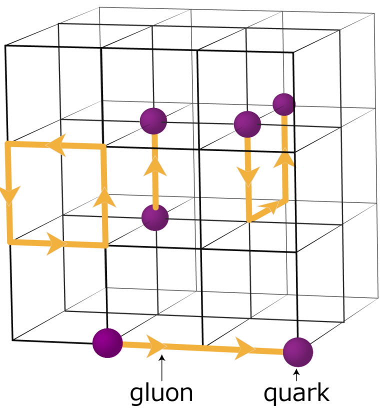


The running of α_s means that there are non-perturbative aspects of QCD.

The non-perturbative aspects can be:

- ⋆ modeled phenomenologically
- ⋆ fitted from experiment
  e.g. factorization:
  $\sigma_{AB} = \sum_{a,b=q,g} \sigma_{ab} \otimes f_{a|A}(x_1, Q^2) \otimes f_{b|B}(x_2, Q^2)$
- ⋆ accessed from first principles
  $\Rightarrow$ LATTICE

QCD and the need for the lattice

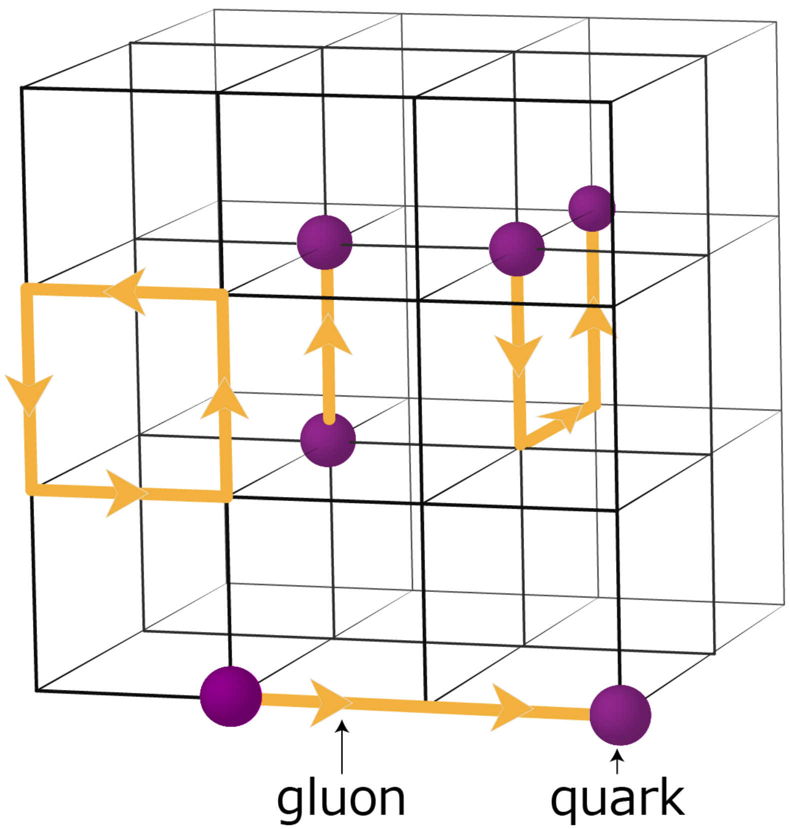


- **Non-perturbative regime of QCD** $\Rightarrow$ quantitative study needs LATTICE.
- **Starting point:** Lagrangian of QCD (*ab initio* method).
- **1st step:** quantize the theory – Euclidean path integral.

  Minkowski path integral can not be used in practice – the phase factor e^{iS} would lead to oscillatory behavior.

  Hence, it is replaced (analytical continuation) by a real-valued exponential e^{-S}.
  Formally, one then evaluates a thermodynamic expectation value with respect to the Boltzmann factor e^{-S}.
- **2nd step:** regularize the theory $\rightarrow$ finite space-time lattice (discretization of the theory).

Lattice formulation



Source: JICFuS, Tsukuba

- We introduce a 4D hypercubic lattice:
  - ⋆ quark fields on lattice sites,
  - ⋆ gluon fields on lattice links.
- Gauge invariant objects:
  - ⋆ Wilson loops,
  - ⋆ quarks and antiquarks connected with a gauge link.
- Lattice as a regulator:
  - ⋆ UV cut-off – inverse lat. spac. a^{-1},
  - ⋆ IR cut-off – inverse lat. size L^{-1}.
- Remove the regulator:
  - ⋆ continuum limit $a \to 0$,
  - ⋆ infinite volume limit $L \to \infty$.

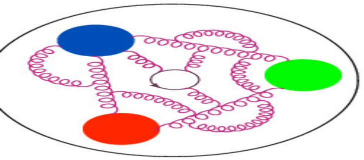
Discretization of the action

- gluonic part – "easy" – gauge action constructed from Wilson loops of size 1x1 (plaquettes) and 1x2 (rectangles):

$$S_G[U] = \frac{\beta}{3} \sum_x \Big(b_0 \sum_{\mu,\nu=1} \mathrm{Re\,Tr}\big(1 - P^{1\times1}_{x;\mu,\nu}\big) + b_1 \sum_{\mu\neq\nu} \mathrm{Re\,Tr}\big(1 - P^{1\times2}_{x;\mu,\nu}\big)\Big),$$

$$\xrightarrow{a\to 0} -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{O}(a^n)$$

where $\beta = 6/g_0^2$, g_0 is the bare coupling and the b_0, b_1 parameters are normalized according to: $b_0 = 1 - 8b_1$.



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  where $\beta = 6/g_0^2$, g_0 is the bare coupling and the b_0, b_1 parameters are normalized according to: $b_0 = 1 - 8b_1$.

- fermionic part – many subtleties and many used discretizations:
  - ⋆ clover fermions,
  - ⋆ twisted mass (TM) fermions
  - ⋆ overlap fermions,
  - ⋆ domain wall fermions,
  - ⋆ staggered fermions,
  - ⋆ other less popular.

The non-uniqueness of discretization offers an opportunity of independent cross-checks!

Simulating QCD on the lattice

- **Lattice QCD can be simulated on a (super)computer!**
- QCD path integral: $Z = \int DU \; e^{-S_{gauge}[U]} \prod_{f=1}^{N_f} \det(\hat{D}_f[U])$.
- Multidimensional integral $\Rightarrow$ Monte Carlo methods.
- How many dimensional integral?
  - ⋆ typical lattice size: $48 \times 48 \times 48 \times 96$ to $96 \times 96 \times 96 \times 192$,
  - ⋆ each lattice site needs 12 spin-color components.

  This gives integral dimension of order 10^8–10^9!
- Huge computational resources needed!
  ... and refined Monte Carlo simulation algorithms!
- 2 main stages:
  - ⋆ generation of gauge ensembles – $\mathcal{O}(100 - 1000)$ Mcore-hours
  - ⋆ calculation of observables – $\mathcal{O}(1 - 1000)$ Mcore-hours

  single-core computer: $\mathcal{O}(10000 - 100000)$ years!

Systematic effects

Ultimately we are interested in continuum QCD.

The power of the lattice approach:

the possibility to control ALL conceivable systematic effects:

- discretization effects $\leftrightarrow$ continuum limit
- finite volume effects $\leftrightarrow$ infinite volume limit
- non-physical quark masses effects $\leftrightarrow$
  chiral extrapolation/simulation with physical masses
- number of active quark flavours $\leftrightarrow$
  simulate desired number of flavours
- isospin breaking $\leftrightarrow$ take $m_u \neq m_d$ + QED into account
- excited states effects $\leftrightarrow$
  optimize operators, increase temporal separation
- renormalization $\leftrightarrow$ non-perturbative procedures
- ...

The degree of control over different systematic effects determines whether one has a *(quantitative) precision study* or a *(qualitative) exploratory study*.

Precision vs. exploratory studies

Two basic kinds of lattice QCD contributions:

- Precision studies
  - ⋆ **quoted errors have fully quantified systematics**,
  - ⋆ some systematics can still be neglected if subleading,
  - ⋆ can be meaningfully (quantitatively) compared to experiment.
- Exploratory studies
  - ⋆ **some important systematics still unknown**,
  - ⋆ in some cases it can be plausibly estimated,
  - ⋆ can be qualitatively compared to experiment, but one needs to keep some things in mind.

Naturally, more difficult problems are longer in the exploratory phase before robust quantitative statements can be made, since difficult problems need time to:

- find the proper way to address
- prove computational feasibility
- optimize the computational method
- acquire all data (long computations...)
- analyze all systematics

Nucleon structure is mostly difficult and expensive computationally!

Hadron structure from lattice QCD

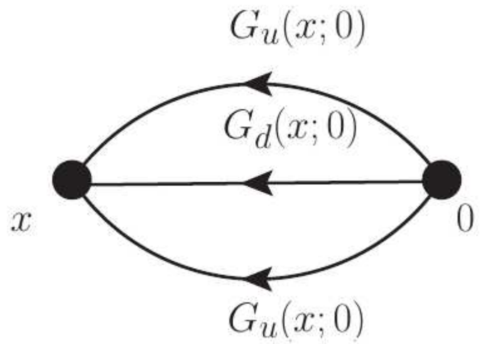
Assume we have a set of gauge configurations and want to extract some hadron structure properties.

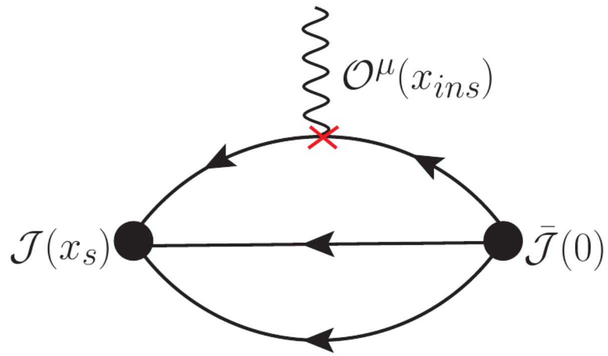
This needs the computation of matrix elements of the generic form:

$$\langle H(t_s)|\mathcal{O}(t_{\text{ins}})|H(t_0)\rangle,$$

where:
$|H(t_0)\rangle$ – hadron state created at some (Euclidean) time t_0,
$|H(t_s)\rangle$ – hadron state annihilated at some time t_s (source-sink separation),
$\mathcal{O}(t_{\text{ins}})$ – operator inserted at some time t_{ins} (insertion time).

Such matrix elements are computed as suitable ratios of 2-point and 3-point correlation functions:





Source: K. Hadjiyiannakou, PhD thesis, Univ. of Cyprus 2015

2-point nucleon correlator

To extract the 2-point nucleon correlator, one needs to excite the nucleon from the vacuum and then annihilate it.

Nucleon interpolating operator:

$$N(x)_\alpha = \mathcal{P}^+ \epsilon^{abc} u(x)^a_\alpha \left(u(x)^b_\beta \left(C\gamma_5\right)_{\beta\gamma} d(x)^c_\gamma \right),$$

$\mathcal{P}^+ = \frac{1+\gamma_0}{4}$ – positive parity projector,

ϵ^{abc} – ensures that the interpolator is colorless and gauge-invariant,

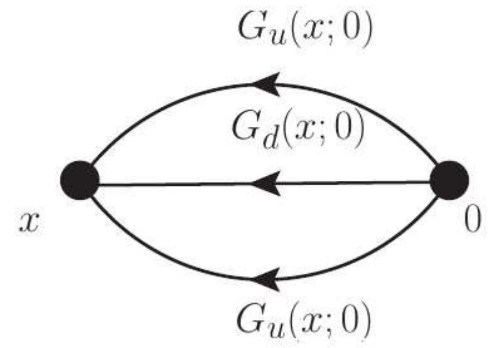
$C = i\gamma_2\gamma_0$ – charge conjugation matrix.

Two-point correlator:

$$\begin{aligned}
C_N(\vec{p},t) &= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \mathcal{P}^+_{\alpha\alpha'} N(\vec{x},t)_\alpha \bar{N}(\vec{0},0)_{\alpha'} \\
&= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \mathcal{P}^+_{\alpha\alpha'} (C\gamma_5)_{\beta\gamma} (C\gamma_5)_{\beta'\gamma'} \epsilon^{abc} \epsilon^{a'b'c'} G_d(x,0)^{cc'}_{\gamma\gamma'} \\
&\qquad \times \left(G_u(x,0)^{bb'}_{\beta\beta'} G_u(x,0)^{aa'}_{\alpha\alpha'} G_u(x,0)^{ab'}_{\alpha\beta'} G_u(x,0)^{ba'}_{\beta\alpha'} \right) \\
&= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \epsilon^{abc} \epsilon^{a'b'c'} G_d(x,0)^{cc'}_{\gamma\gamma'} \\
&\qquad \times \Bigg[\mathrm{Tr}\left(\left(C\gamma_5 G_d(x,0) (C\gamma_5)^T \right)^{cc'} \left(G_u^T(x,0) \right)^{bb'} \right) \mathrm{Tr}\left(\left(G_u(x,0)\mathcal{P}^+ \right)^{aa'} \right) \\
&\qquad - \mathrm{Tr}\left(\left(\left(C\gamma_5 G_d(x,0) (C\gamma_5)^T \right)^T \right)^{cc'} \left(G_u(x,0) \right)^{ab'} \left(\mathcal{P}^+ G_u(x,0) \right)^{ba'} \right) \Bigg)
\end{aligned}$$

2-point nucleon correlator

Hence, to compute the 2-point correlator, one basically needs to know the **quark propagator from the source to all sink points**.
$\longrightarrow$ most costly part of the calculation.



One needs to invert the Dirac operator, i.e. solve $D\psi = \eta$, where D is of dimension $10^8 - 10^9$.

In the Heisenberg picture, $N(\vec{x},t) = e^{-\vec{P}\cdot\vec{x}} e^{Ht} N(\vec{0},0) e^{-Ht} e^{i\vec{P}\cdot\vec{x}}$.
Inserting a complete set of states, $\sum_{\vec{p},n} |n,\vec{p}\rangle\langle n,\vec{p}|$, we obtain the zero-momentum correlator:

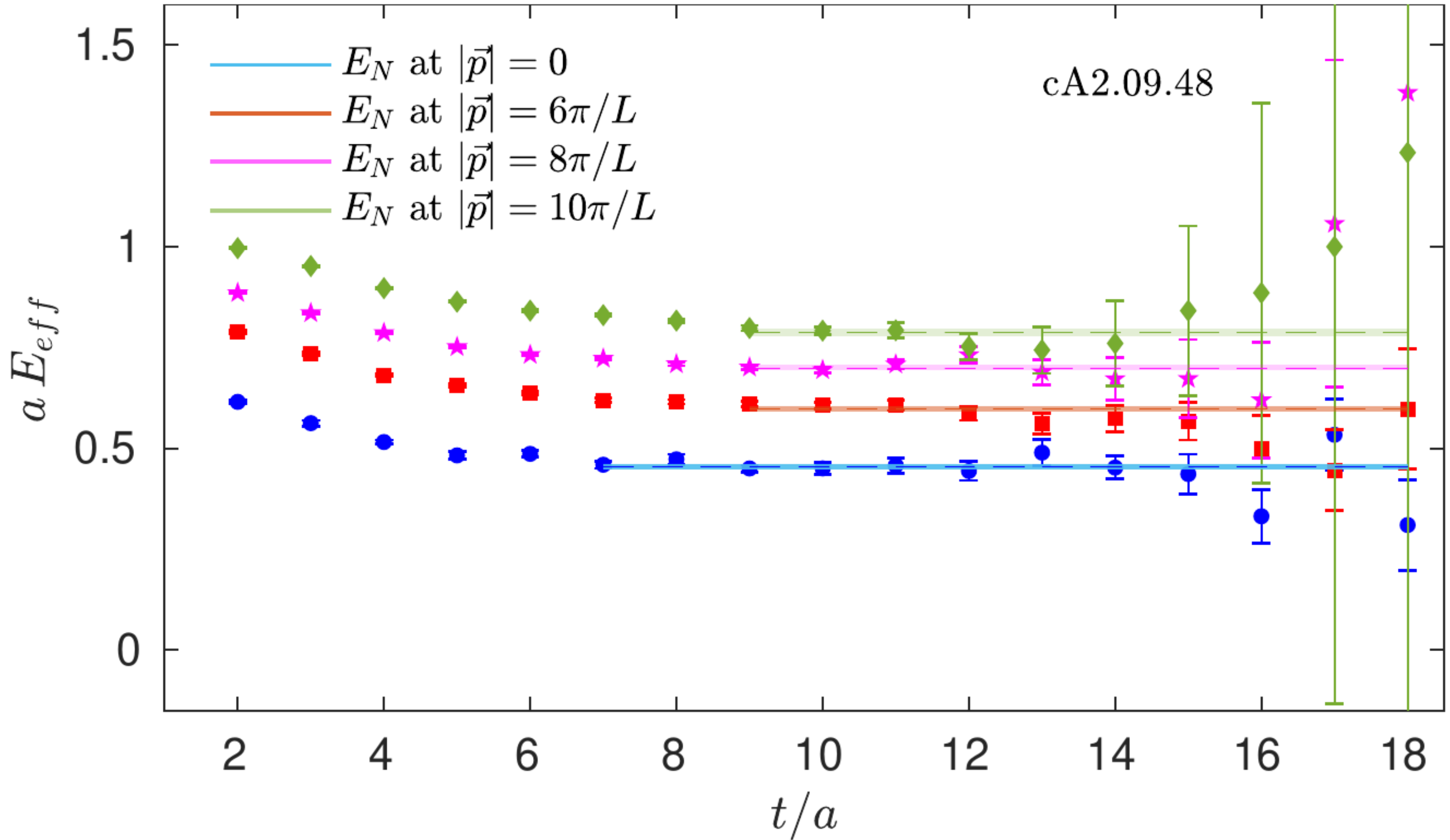
$$C_N(t) = \sum_{\vec{p},n}\sum_{\vec{x}} \left|\langle 0|N(\vec{0},0)|n,\vec{p}\rangle\right|^2 e^{-E_n(\vec{p})t} e^{i\vec{p}\cdot\vec{x}}$$
$$= \sum_n Z_0 e^{-E_0 t}\left(1 + Z_1 e^{-\Delta E_1 t} + Z_2 e^{-\Delta E_2 t} + \ldots\right),$$

where $Z_n = \left|\langle 0|N(\vec{0},0)|n,\vec{0}\rangle\right|^2$ and $\Delta E_n = E_n - E_0$.

We have excited all baryons with quantum numbers of the nucleon!

2-point nucleon correlator

To suppress excited states, one needs to go to large Euclidean times:



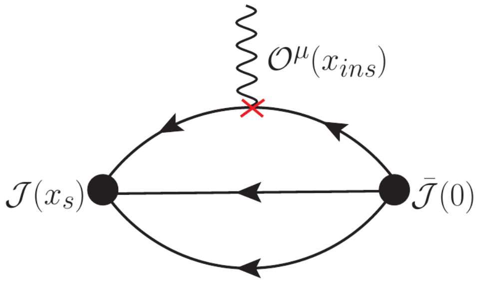
Source: A. Scapellato, Ph.D. thesis, Univ. of Cyprus 2019

When boosting the nucleon:

- excited states contamination worsens,
- signal-to-noise ratio decreases ($N_{\mathrm{meas}} = 320, 9504, 38250, 72990$ in the plot).

3-point nucleon correlator

Analogously, one can extract the 3-point correlator.
BUT: one needs to vary the **operator insertion** point and thus, one needs special techniques to calculate the **all-to-all quark propagator**.



To have excited states under control, one needs to keep large separations:

- from the source to the sink – $|t_s - t_0| \gg 0$,
- from the source to the insertion point – $|t_{\rm ins} - t_0| \gg 0$,
- from the sink to the insertion point – $|t_s - t_{\rm ins}| \gg 0$.

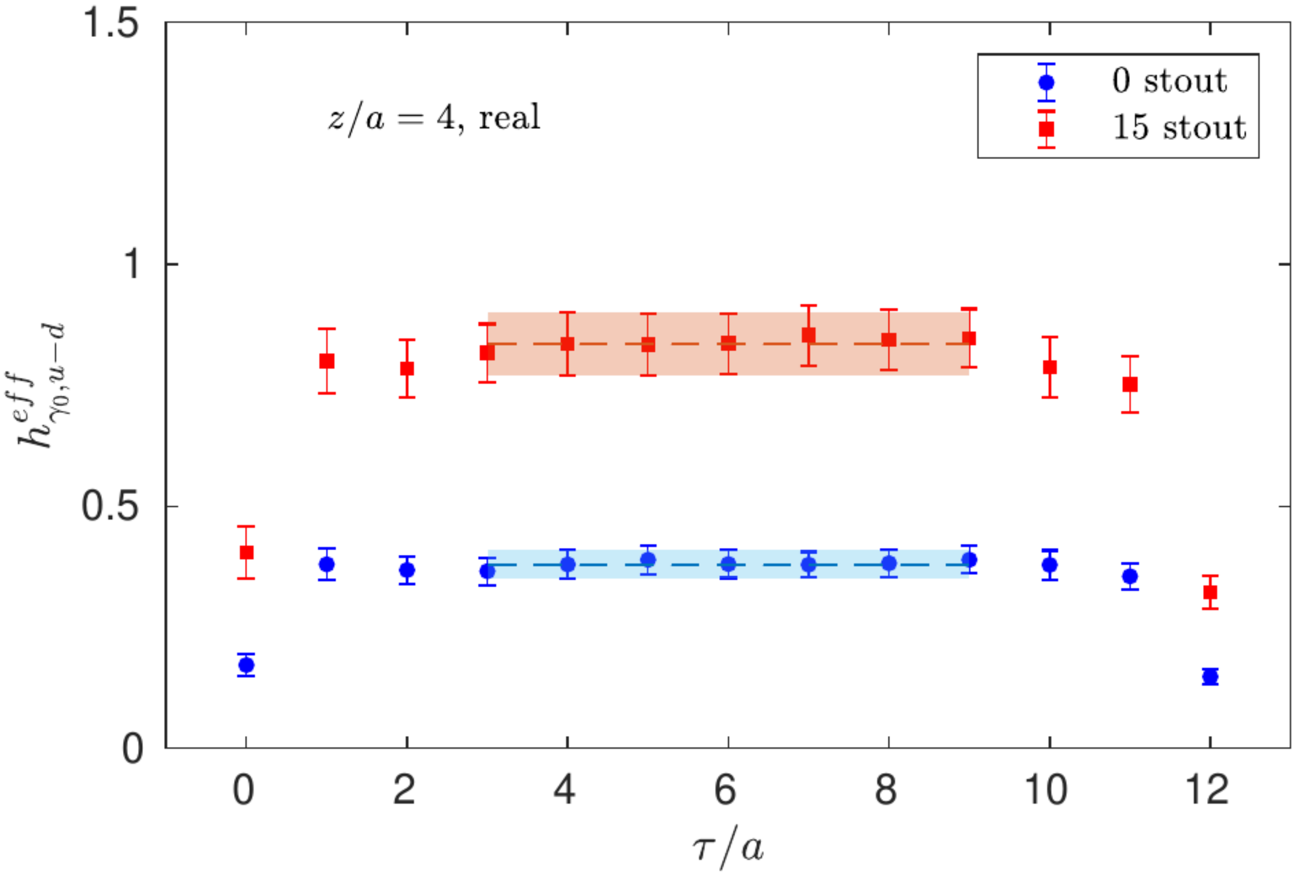
If these conditions are satisfied, one can form a ratio and extract the desired matrix element:

$$\langle H(t_s)|\mathcal{O}(t_{\rm ins})|H(t_0)\rangle = \mathcal{K}\frac{C_{\rm 3pt}(t_0, t_s, t_{\rm ins})}{C_{\rm 2pt}(t_s - t_0)},$$

$\mathcal{K}$ – kinematic factor.

3-point nucleon correlator

Example of thus extracted matrix element:



Source: A. Scapellato, Ph.D. thesis, Univ. of Cyprus 2019

- Good suppression of excited states indicated by the plateau in $t_{\rm ins}$.
- Still, one needs to carefully check the dependence on $t_s - t_0$!

Renormalization

Matrix elements computed on the lattice are **finite**, because lattice serves as an IR and UV regulator.

However, taking a continuum limit one would in many cases observe that they **diverge**.

Hence, they require **renormalization**.

Options:

- $\overline{\text{MS}}$ – NO! Can't use dimensional regularization...
- perturbative renormalization – YES, but undesirable,
- non-perturbative renormalization – PREFERRED WAY.

Options for non-perturbative renormalization:

- Schrödinger functional,
- gradient flow,
- Ward identities,
- **regularization-independent momentum subtraction (RI/MOM)**,
- coordinate space (X-space) scheme,
- cancel divergences with a suitable ratio.

Renormalization factors perturbatively converted to the $\overline{\text{MS}}$ scheme.

RI/MOM renormalization

Renormalization conditions (so-called RI' variant):
for the operator:

$$Z_q^{-1}\, Z_{\mathcal{O}}\, \frac{1}{12}\mathrm{Tr}\left[\mathcal{V}(p)\left(\mathcal{V}^{\mathrm{Born}}(p)\right)^{-1}\right]\Big|_{p^2=\bar{\mu}_0^2} = 1\,,$$

for the quark field:

$$Z_q = \frac{1}{12}\mathrm{Tr}\left[(S(p))^{-1}\, S^{\mathrm{Born}}(p)\right]\Big|_{p^2=\bar{\mu}_0^2}\,.$$

- momentum p in the vertex function is set to the RI′ renormalization scale $\bar{\mu}_0$
- $\mathcal{V}(p)$ – amputated vertex function of the operator,
- $\mathcal{V}^{\mathrm{Born}}$ – its tree-level value, e.g. $\mathcal{V}^{\mathrm{Born}}(p){=}i\gamma_\mu\gamma_5$ for the axial current,
- $S(p)$ – fermion propagator ($S^{\mathrm{Born}}(p)$ at tree-level).

This prescription handles all divergences that are present and applies the necessary finite renormalization related to the lattice regularization.

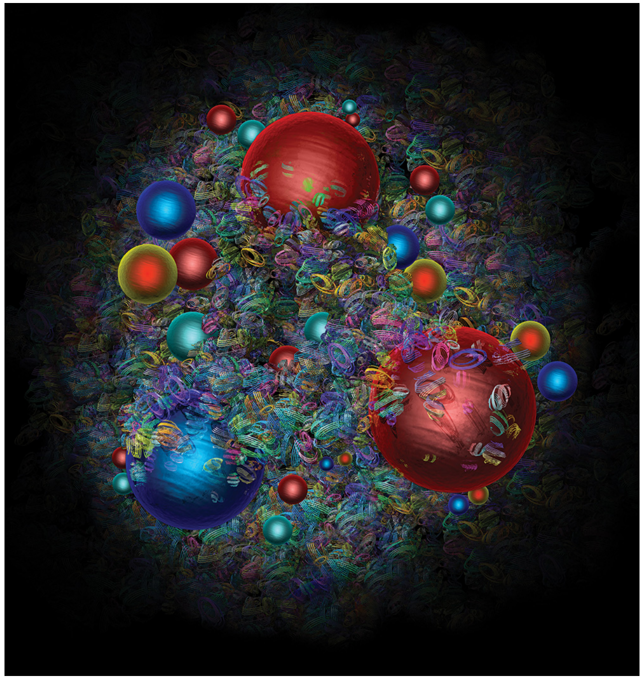
EIC's central aim is to understand better nucleon's internal structure.

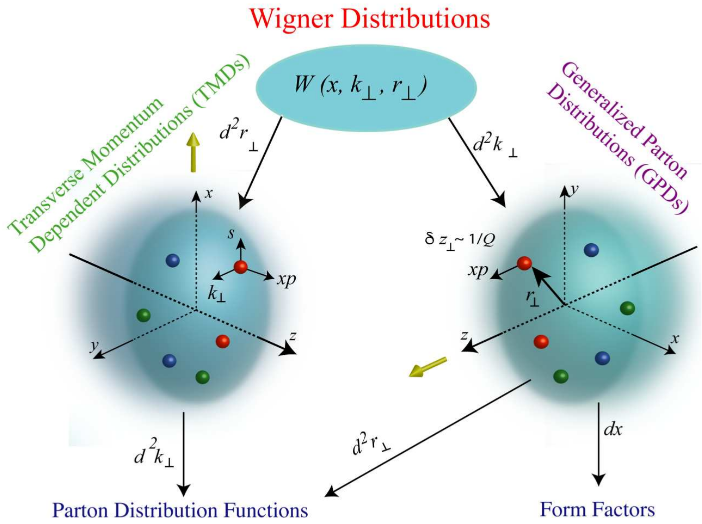
This has various aspects:

- how the quarks and gluons move inside the nucleon,
- 3D imaging of the nucleon – "hadron tomography",
- role of gluons and their emergent properties,
- how is spin decomposed,
- origin of nucleon mass,
- ...

Lattice can provide *qualitative* and eventually *quantitative* knowledge of different functions and their moments:

- 1D: form factors
- 1D: parton distribution functions (PDFs)
- 3D: generalized parton distributions (GPDs)
- 3D: transverse momentum dependent PDFs (TMDs)
- 5D: Wigner function





Moments of PDFs/GPDs on the lattice

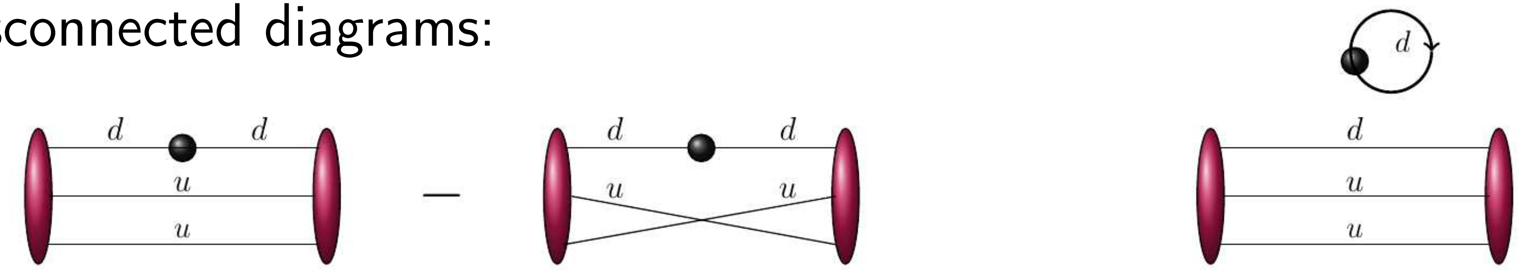
Moments of PDFs/GPDs are defined via matrix elements of local operators:

$$\int dx\, x^{n-1} f(x, Q^2, \mu^2) \propto \langle N(p', s')|\mathcal{O}^{\{\mu_1 \ldots \mu_n\}}|N(p, s)\rangle\Big|_{\mu^2},$$

$|N(p, s)\rangle$ – nucleon with momentum p and spin s,

operator: $\mathcal{O}^{\{\mu_1 \ldots \mu_n\}} = \bar{\psi}\left(\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}}\right) \frac{\tau^a}{2} \psi.$

Important: flavor structure determined by the τ matrix above.
Flavor-decomposed quantities require both quark-connected and disconnected diagrams:



Source: A. Scapellato, Ph.D. thesis, Univ. of Cyprus 2019

With degenerate light quarks (standard treatment so far), disconnected contributions cancel in isovector quantities, i.e. $u - d$.

Case 1. Nucleon charges

$$\int dx\, x^{n-1} f(x, Q^2, \mu^2) \propto \langle N(p', s')|\mathcal{O}^{\{\mu_1 \ldots \mu_n\}}|N(p, s)\rangle\big|_{\mu^2},$$

$|N(p,s)\rangle$ – nucleon with momentum p and spin s,

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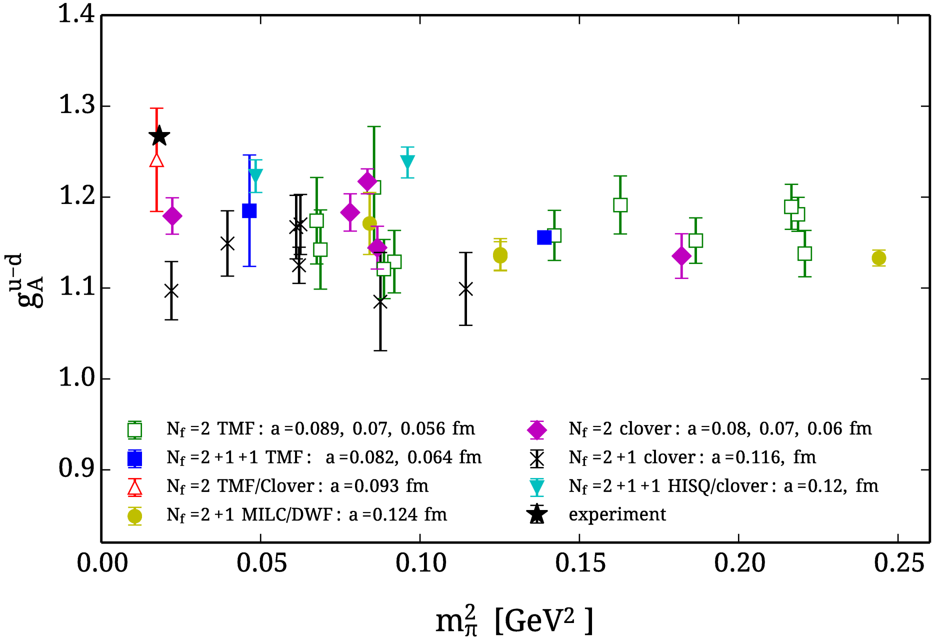
Simplest case: $n = 1$, no momentum transfer $p = p'$.

- $\mathcal{O} = \gamma^\mu$ – vector $\rightarrow$ trivial (vector current conservation),
- $\mathcal{O} = \gamma^\mu \gamma^5$ – axial charge g_A,
- $\mathcal{O} = \sigma^{\mu\nu}$ – tensor charge g_T,
- $\mathcal{O} = 1$ – scalar charge g_S
  σ-terms: $m_f \langle N|\overline{\psi}_f \psi|N\rangle$.

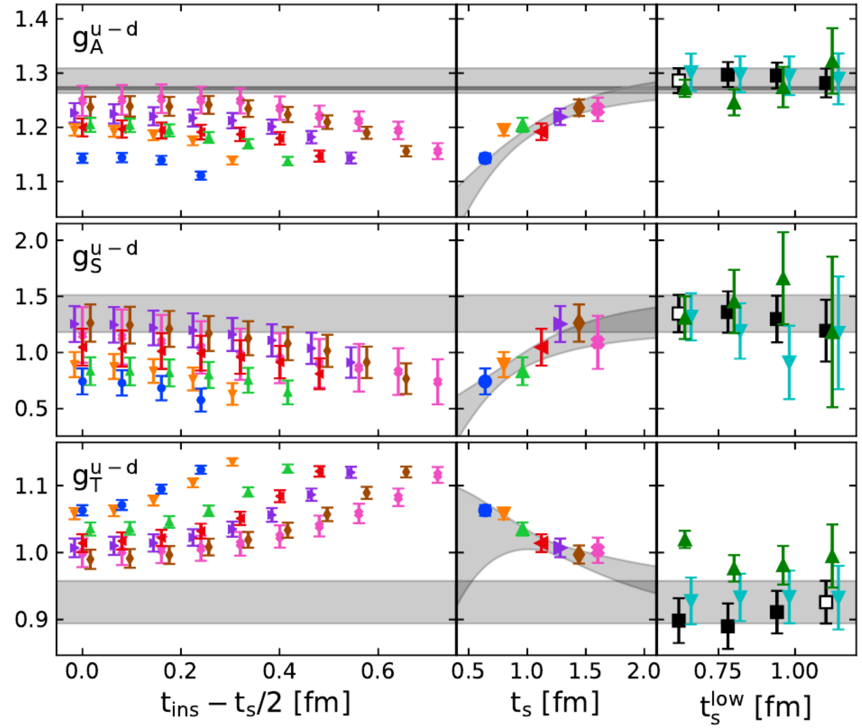
The isovector charges can be calculated very precisely on the lattice, with fully controlled systematics and total error at the percent level.

The flavor-decomposed ones more demanding computationally, but already within reach with $\mathcal{O}(10\%)$ precision.

Actually, it was a long story to reproduce the experimentally well-determined g_A^{u-d}:



ETMC, PRD92(2015)114513



ETMC, PRD102(2020)054517

Many systematic effects needed to be understood.

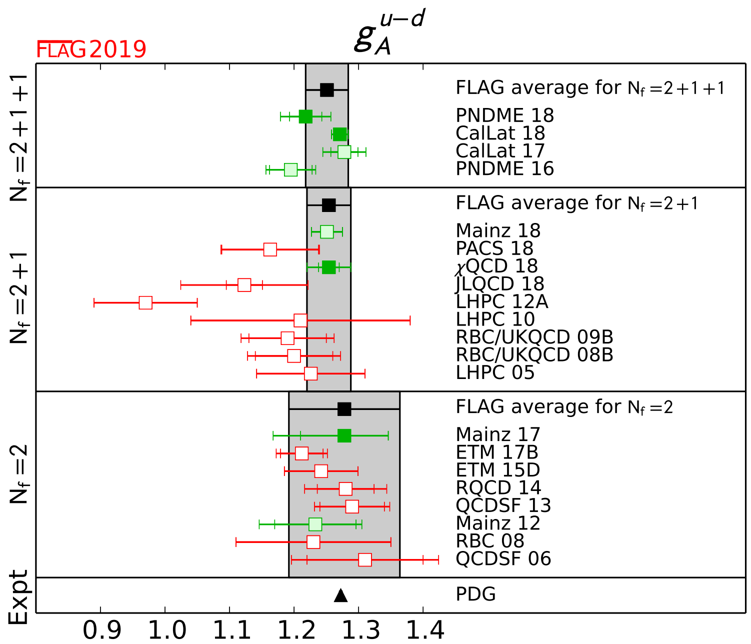
In particular, control over excited states turned out to be crucial.

Nucleon axial charge

FLAG19 review S. Aoki et al., Eur.Phys.J.C80(2020)113, 1902.08191

Flavour non-singlet

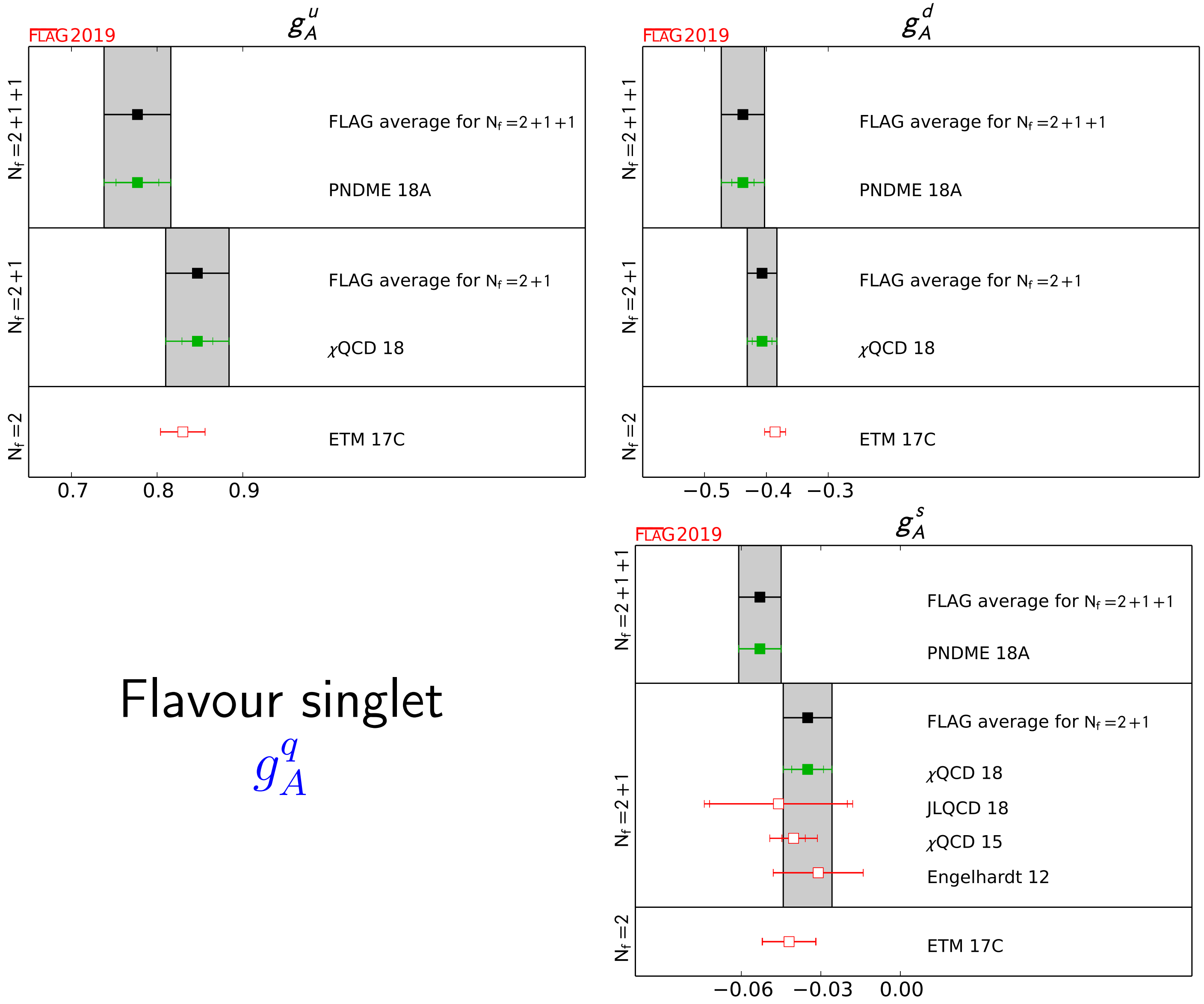
g_A^{u-d}



Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	g_A^{u-d}
PNDME 18[a]	[84]	2+1+1	A	★‡	★	★	★	★	1.218(25)(30)
CalLat 18	[85]	2+1+1	A	○	★	★	★	★	1.271(10)(7)
CalLat 17	[830]	2+1+1	P	○	★	★	★	★	1.278(21)(26)
PNDME 16[a]	[829]	2+1+1	A	○‡	★	★	★	★	1.195(33)(20)
Mainz 18	[914]	2+1	C	★	○	★	★	★	1.251(24)
PACS 18	[807]	2+1	A	■	■	★	★	■	1.163(75)(14)
χQCD 18	[6]	2+1	A	○	★	★	★	★	1.254(16)(30)$
JLQCD 18	[838]	2+1	A	■	○	○	★	★	1.123(28)(29)(90)
LHPC 12A[b]	[915]	2+1	A	■‡	★	★	★	★	0.97(8)
LHPC 10	[845]	2+1	A	■	○	■	★	■	1.21(17)
RBC/UKQCD 09B	[832]	2+1	A	■	■	○	★	■	1.19(6)(4)
RBC/UKQCD 08B	[831]	2+1	A	■	■	○	★	■	1.20(6)(4)
LHPC 05	[916]	2+1	A	■	■	★	★	■	1.226(84)
Mainz 17	[86]	2	A	★	★	★	★	○	$1.278(68)(^{+0}_{-0.087})$
ETM 17B	[823]	2	A	■	○	○	★	★	1.212(33)(22)
ETM 15D	[821]	2	A	■	○	○	★	★	1.242(57)
RQCD 14	[818]	2	A	○	★	★	★	■	1.280(44)(46)
QCDSF 13	[399]	2	A	○	★	■	★	■	1.29(5)(3)
Mainz 12	[817]	2	A	★	○	○	★	○	$1.233(63)(^{+0.035}_{-0.060})$
RBC 08	[917]	2	A	■	■	■	★	■	1.23(12)
QCDSF 06	[816]	2	A	○	■	■	★	■	1.31(9)(7)

Nucleon axial charge

FLAG19 review S. Aoki et al., Eur.Phys.J.C80(2020)113, 1902.08191

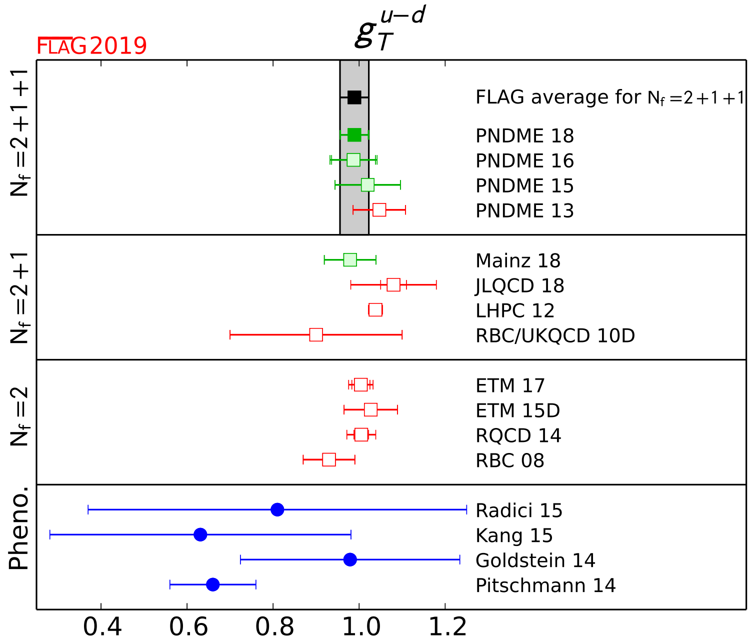


Flavour singlet

g_A^q

Nucleon tensor charge

FLAG19 review S. Aoki et al., Eur.Phys.J.C80(2020)113, 1902.08191



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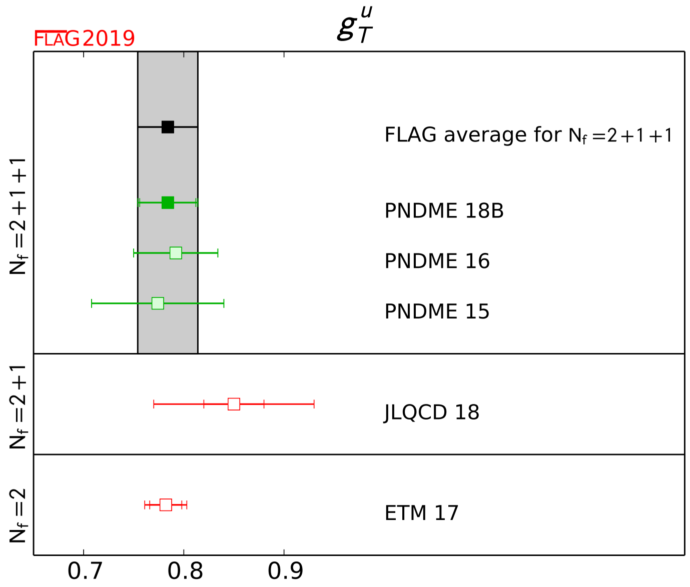
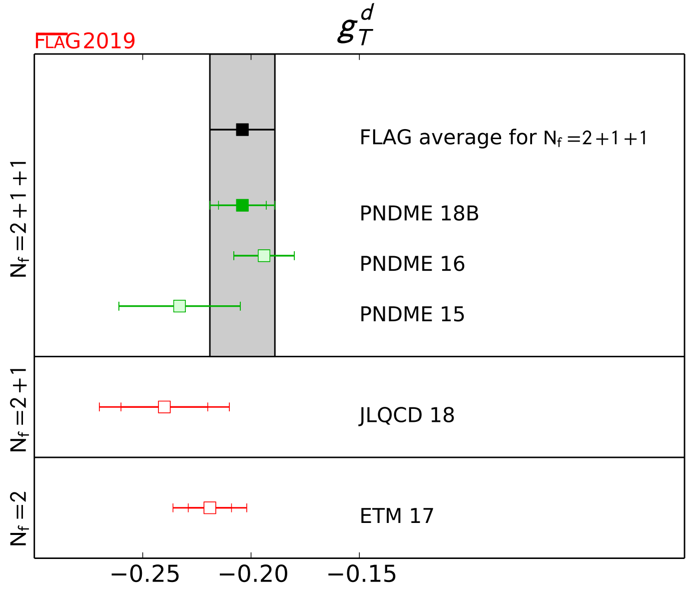
PNDME 18	[84]	2+1+1	A	★‡	★	★	★	★	0.989(32)(10)
PNDME 16	[829]	2+1+1	A	○‡	★	★	★	★	0.987(51)(20)
PNDME 15	[827, 828]	2+1+1	A	○‡	★	★	★	★	1.020(76)
PNDME 13	[826]	2+1+1	A	■‡	■	★	★	★	1.047(61)
Mainz 18	[914]	2+1	C	★	○	★	★	★	0.979(60)
JLQCD 18	[838]	2+1	A	■	○	○	★	★	1.08(3)(3)(9)
LHPC 12	[919]	2+1	A	■‡	★	★	★	★	1.038(11)(12)
RBC/UKQCD 10D	[833]	2+1	A	■	■	○	★	■	0.9(2)
ETM 17	[825]	2	A	■	○	○	★	★	1.004(21)(2)(19)
ETM 15D	[821]	2	A	■	○	○	★	★	1.027(62)
RQCD 14	[818]	2	A	○	★	★	★	■	1.005(17)(29)
RBC 08	[917]	2	A	■	■	■	★	■	0.93(6)

Flavour non-singlet

g_T^{u-d}

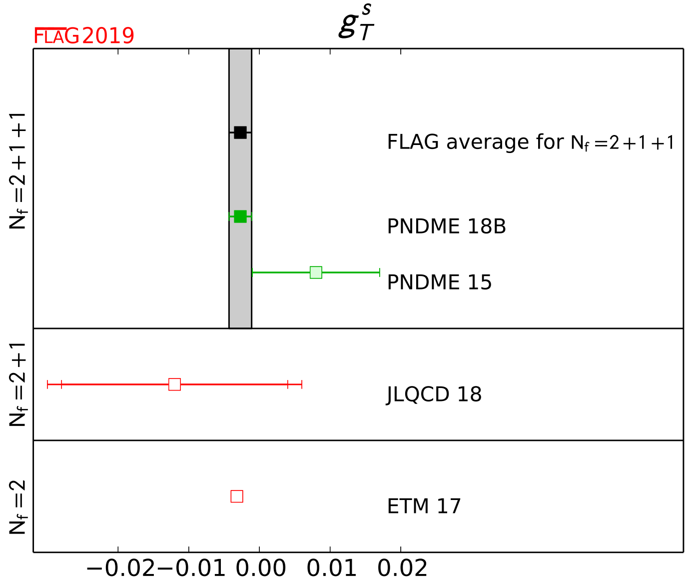
FLAG19 review S. Aoki et al., Eur.Phys.J.C80(2020)113, 1902.08191





Flavour singlet

g_T^q



Case 2. Form factors

$$\int dx\, x^{n-1} f(x,Q^2,\mu^2) \propto \langle N(p',s')|\mathcal{O}^{\{\mu_1\ldots\mu_n\}}|N(p,s)\rangle\big|_{\mu^2},$$

$|N(p,s)\rangle$ – nucleon with momentum p and spin s,

operator: $\mathcal{O}^{\{\mu_1\ldots\mu_n\}} = \bar{\psi}\left(\gamma^{\{\mu_1} i\overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\right)\frac{\tau^a}{2}\psi$.

Case 2: $n=1$, with momentum transfer Q^2 ($p \neq p'$).

Electromagnetic form factors:

$$\langle N(p',s')|\overline{\psi}\gamma^\mu\psi|N(p,s)\rangle = \bar{u}(p',s')\left(\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}Q_\nu}{2m_N}F_2(Q^2)\right)\frac{1}{2}u(p,s),$$

$F_1(Q^2)$ – Dirac FF, $F_2(Q^2)$ – Pauli FF

Sachs FFs: $G_E = F_1 + (Q^2/4m_N^2)F_2$, $G_M = F_1 + F_2$.

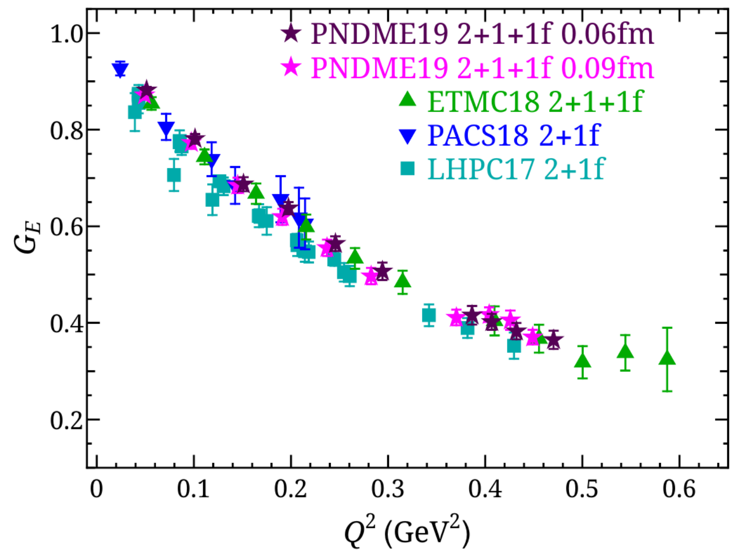
Axial form factors:

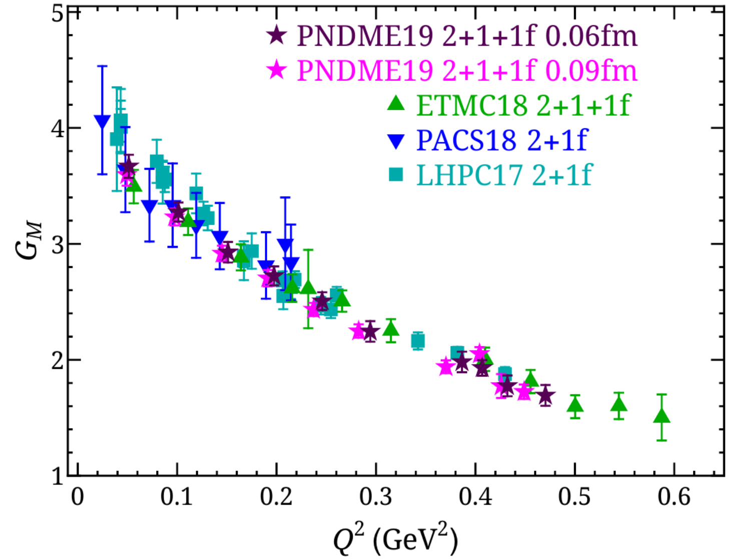
$$\langle N(p',s')|\overline{\psi}\gamma^\mu\gamma^5\psi|N(p,s)\rangle = \bar{u}(p',s')\left(\gamma^\mu\gamma^5 G_A(Q^2) + \frac{\gamma^5 Q^\mu}{2m_N}G_P(Q^2)\right)\frac{1}{2}u(p,s),$$

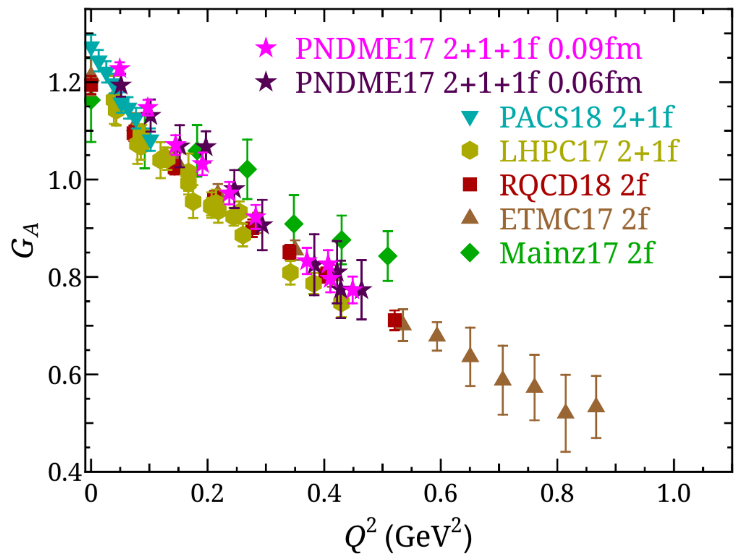
$G_A(Q^2)$ – axial FF, $G_P(Q^2)$ – induced pseudoscalar FF

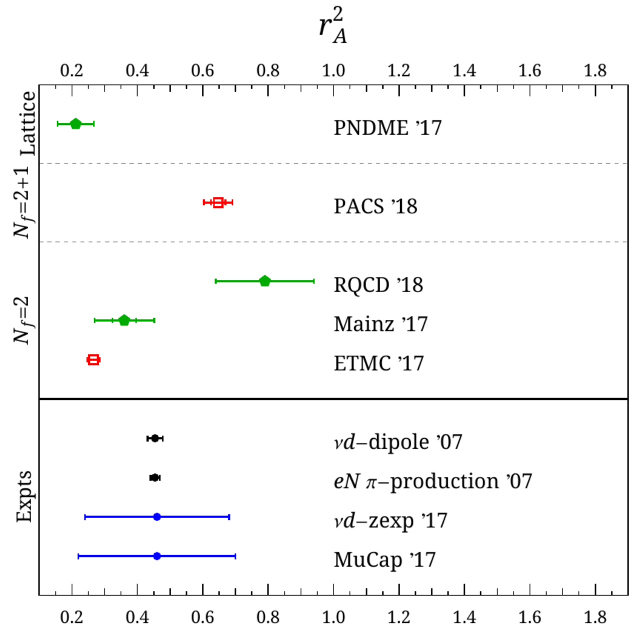
Form factors and axial radius

PDFLattice20 review M. Constantinou et al., 2006.08636









Case 3. Generalized form factors

$$\int dx\, x^{n-1} f(x, Q^2, \mu^2) \propto \langle N(p', s')|\mathcal{O}^{\{\mu_1 \ldots \mu_n\}}|N(p,s)\rangle\big|_{\mu^2},$$

$|N(p,s)\rangle$ – nucleon with momentum p and spin s,

operator: $\mathcal{O}^{\{\mu_1 \ldots \mu_n\}} = \bar{\psi}\left(\gamma^{\{\mu_1} i\overleftrightarrow{D}^{\mu_2} \ldots i\overleftrightarrow{D}^{\mu_n\}}\right)\frac{\tau^a}{2}\psi.$

Case 3: $n = 2$ (one-derivative operators), with momentum transfer Q^2 $(p \neq p')$, average momentum $P = (p + p')/2$.

Vector operator:

$$\langle N(p', s')|\overline{\psi}\gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}}\psi|N(p,s)\rangle = \bar{u}(p', s')\Bigg(A_{20}(Q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}(Q^2)\frac{i\sigma^{\{\mu\alpha}Q_\alpha P^{\nu\}}}{2m_N} + C_{20}(Q^2)Q^{\{\mu}Q^{\nu\}}\Bigg)\frac{1}{2}u(p,s),$$

Axial operator:

$$\langle N(p', s')|\overline{\psi}\gamma^5\gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}}\psi|N(p,s)\rangle = \bar{u}(p', s')\Bigg(\tilde{A}_{20}(Q^2)\gamma^{\{\mu}P^{\nu\}}\gamma^5 + \tilde{B}_{20}(Q^2)\frac{Q^{\{\mu}P^{\nu\}}}{2m_N}\gamma^5\Bigg)\frac{1}{2}u(p,s).$$

Nucleon spin decomposition

Moments/FFs defined in previous slides can be used to decompose nucleon angular momentum to contributions from different partons!

Ji's (gauge-invariant) decomposition: X. Ji, PRL78(1997)610

$$\underbrace{J_N}_{\text{TOTAL}} = \underbrace{J_g}_{\text{gluon AM}} + \sum_q \left(\underbrace{\frac{1}{2}\Delta\Sigma_q}_{\text{quark spin}} + \underbrace{L_q}_{\text{quark OAM}} \right).$$

Contributions:

- $J_{q/g} = \frac{1}{2}\left[A_{20}^{q/g}(0) + B_{20}^{q/g}(0)\right]$ – unpolarized GFFs in the forward limit,
- $A_{20}^{q/g}(0) = \langle x \rangle_{q/g}$ – can be computed directly at zero momentum transfer,
- but $B_{20}^{q/g}(0)$ need to be accessed as $Q^2 \to 0$ limit of GFFs,
- decomposing quark J_q further: quark spin from $\Delta\Sigma_q = g_A^q$, quark OAM – indirectly as $L^q = J^q - \frac{1}{2}\Delta\Sigma_q$.

Nucleon spin decomposition

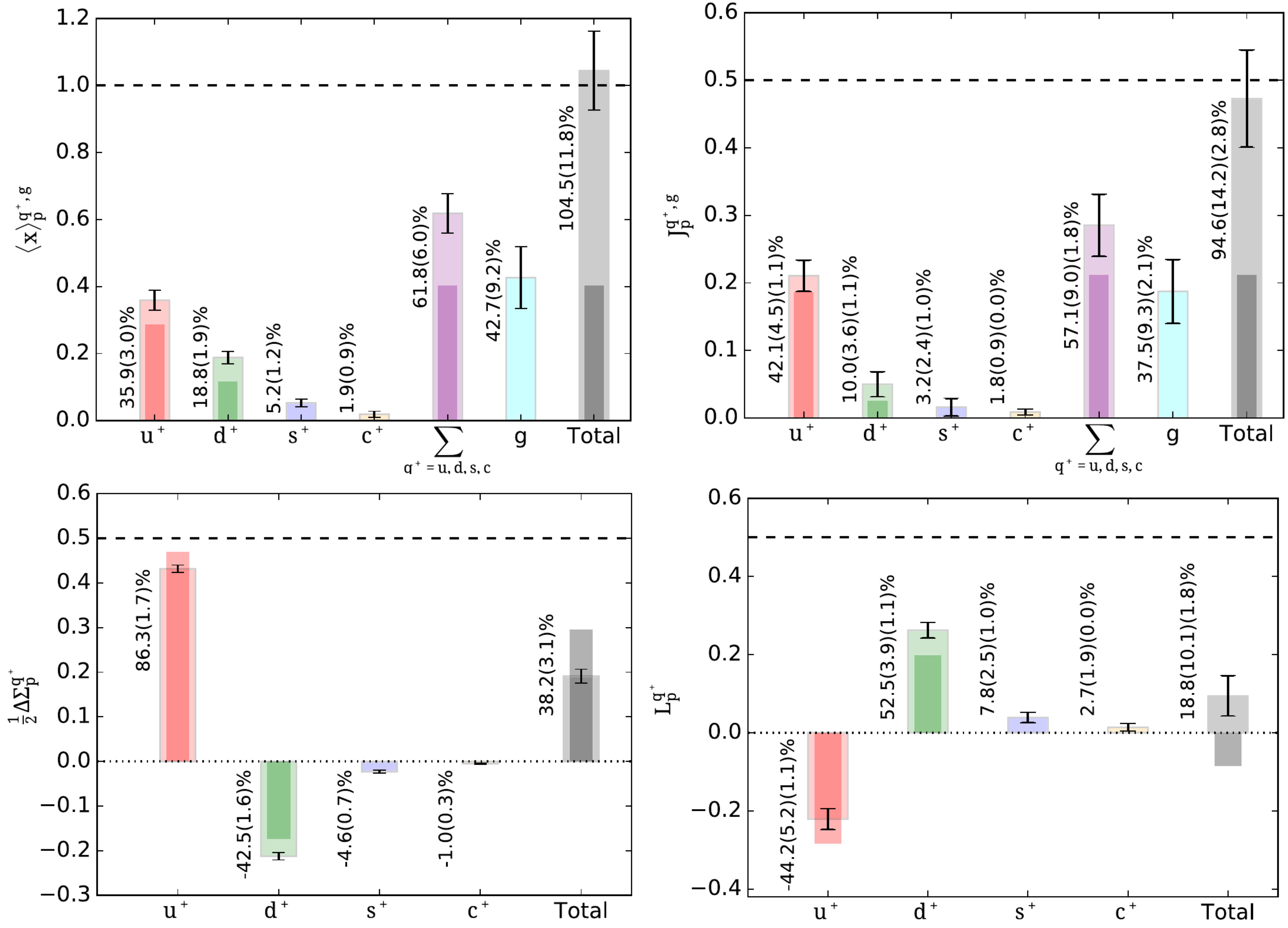
Summary of what needs to be computed on the lattice:

- Quark and gluon momentum fractions – $\langle x \rangle_{q/g}$
  separately for each quark flavor and for gluons → quark-disconnected diagrams.
- $B_{20}^{q/g}(Q^2)$ for several values of Q^2 to reliably extrapolate to the forward limit
  one-derivative operators, again separately for each parton.
- Flavor decomposition of the axial charge g_A^q
  simple MEs of the axial current, but again quark-disconnected diagrams.

All of these require very non-trivial renormalization – mixing between quarks and gluons!

Nucleon spin decomposition

ETMC, PRD101(2020)094513, PRL119(2017)142002



x-dependent distributions

As we have seen, lattice is quite successful in computing moments of PDFs/GPDs.

If we are interested in full x-dependent partonic distributions, we can just calculate higher moments and reconstruct the distributions.

WRONG!!!

Higher moments are basically inaccessible, because:

- operators with more derivatives are very noisy,
- there is inevitable operator mixing with lower-dimensional operators.

Thus, one needs other approaches to access the x-dependence.

Actually, they have recently become available and are being very actively pursued.

x-dependent distributions

Definition of PDFs:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0-\xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-,0)$ is the Wilson line from 0 to ξ^-.

It is given in terms of non-local light-cone correlators – intrinsically Minkowskian – **problem for the lattice!**

We say it is light-cone dominated – i.e. needs $\xi^2 = \vec{x}^2 + t^2 \sim 0$ – very hard due to non-zero lattice spacing!

Thus, an alternative approach is needed, formulated in terms of Euclidean matrix elements.

Approaches to x-dependence

- Recent years (since ≈ 2013): breakthrough in accessing x-dependence.
- The common feature of all the approaches is that they rely to some extent on the factorization framework:

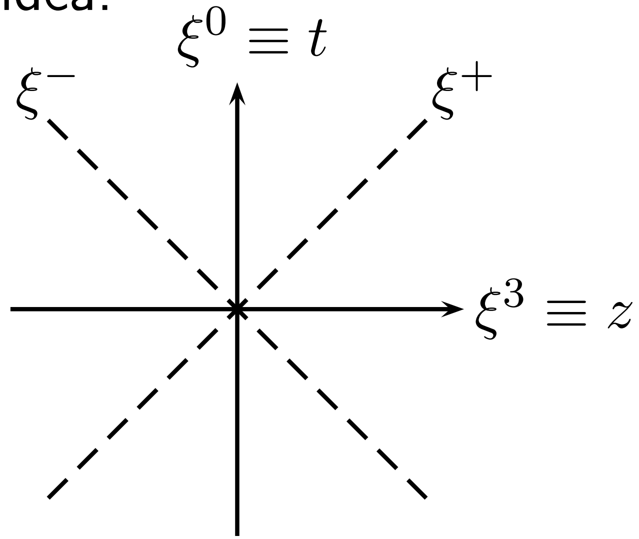
$$\underbrace{Q(x,\mu_R)}_{\text{some lattice observable}} = \int_{-1}^{1} \frac{dy}{y}\, C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$
with different choices of Γ,Γ' Dirac structures and objects $F(z)$.
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

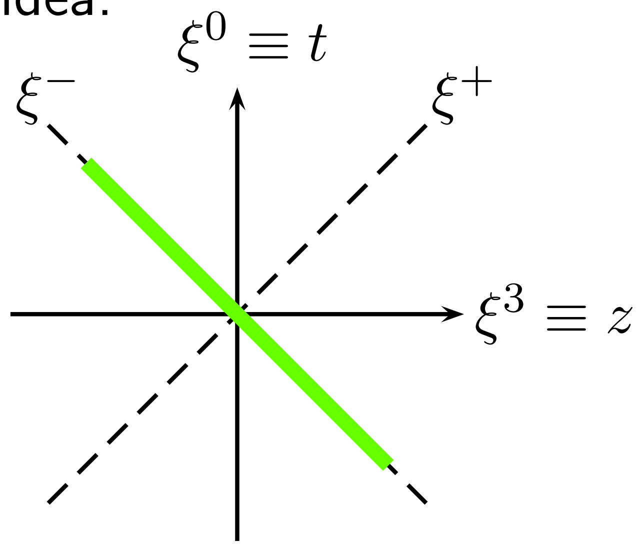
Main idea:



Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

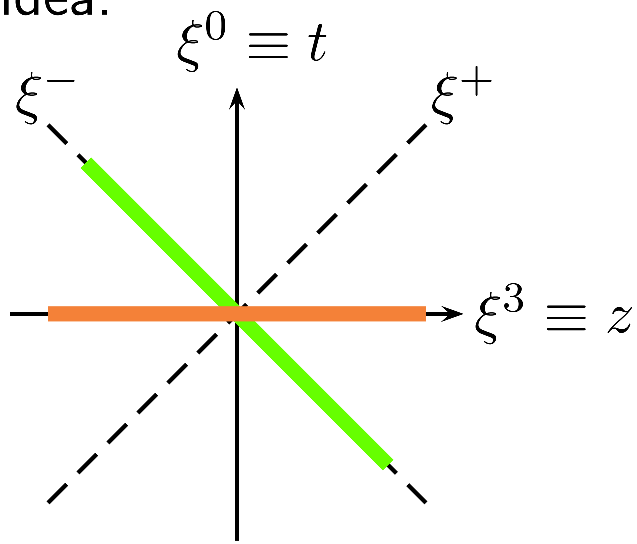
$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma \mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

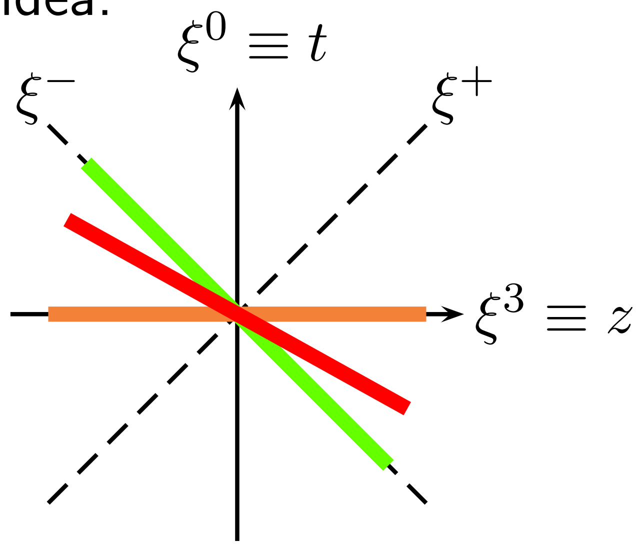
$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

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Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

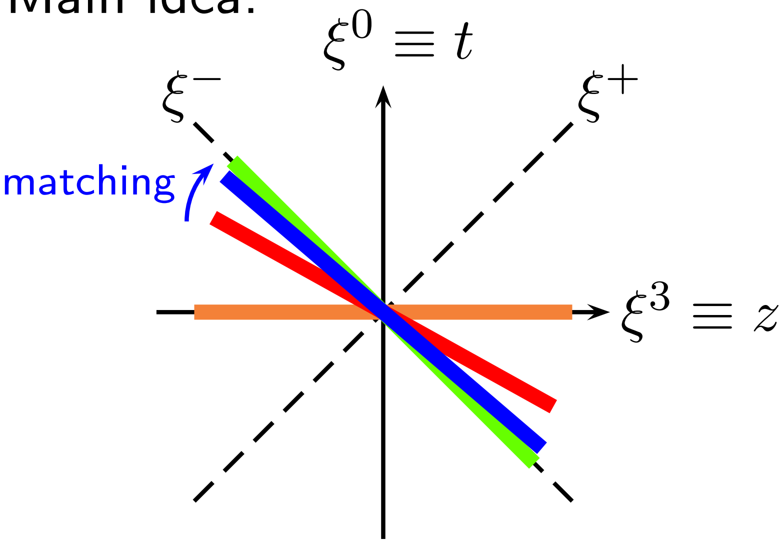
$|P\rangle$ – boosted nucleon

Quasi-PDFs

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

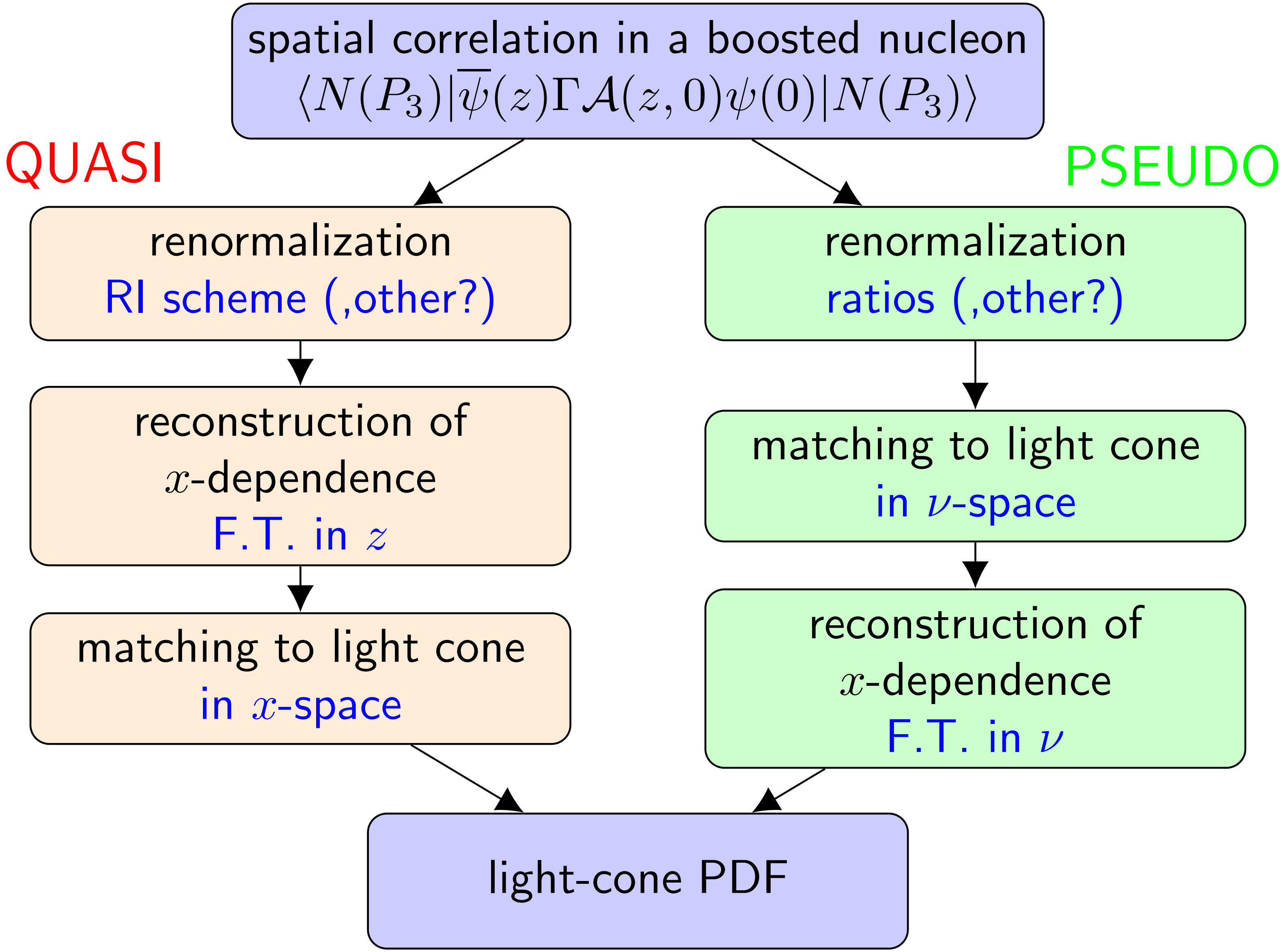
Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\underbrace{\tilde{q}(x,\mu,P_3)}_{\text{quasi-PDF}} = \int_{-1}^{1} \frac{dy}{|y|} \underbrace{C\left(\frac{x}{y}, \frac{\mu}{P_3}\right)}_{\text{pert.kernel}} \underbrace{q(y,\mu)}_{\text{PDF}} + \underbrace{\mathcal{O}\left(\Lambda^2_{\text{QCD}}/P_3^2, M_N^2/P_3^2\right)}_{\text{higher-twist effects}}$$

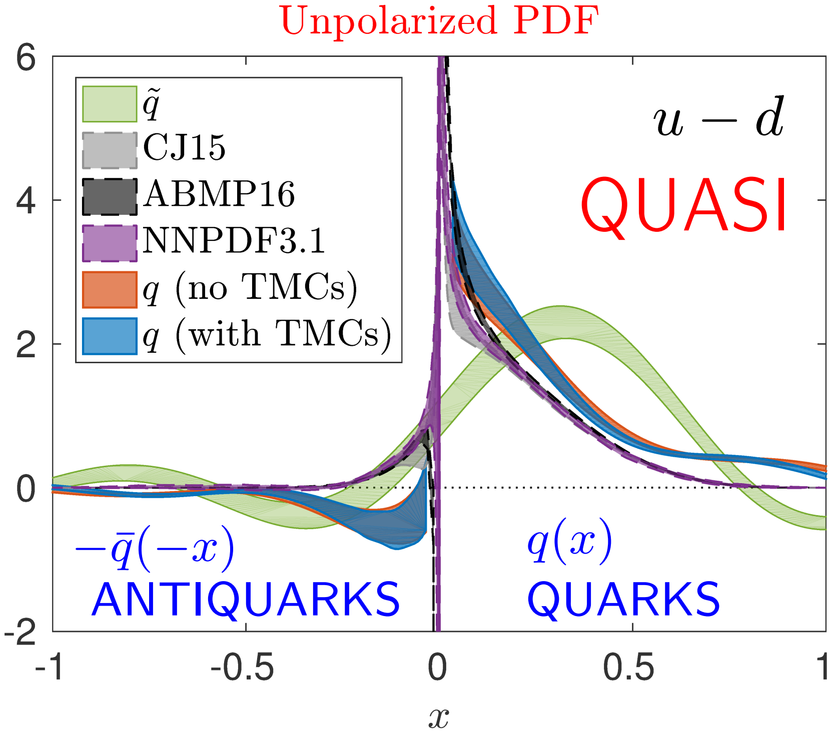
Quasi-PDFs vs. pseudo-PDFs



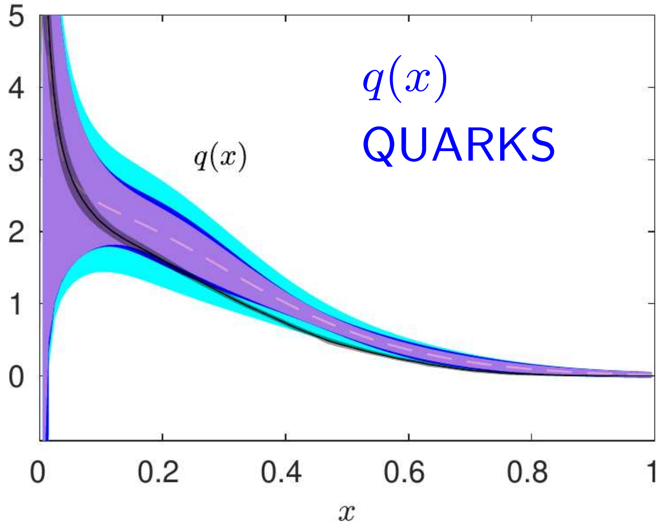
ETMC, Phys. Rev. Lett. 121 (2018) 112001
ETMC, Phys. Rev. D 99 (2019) 114504

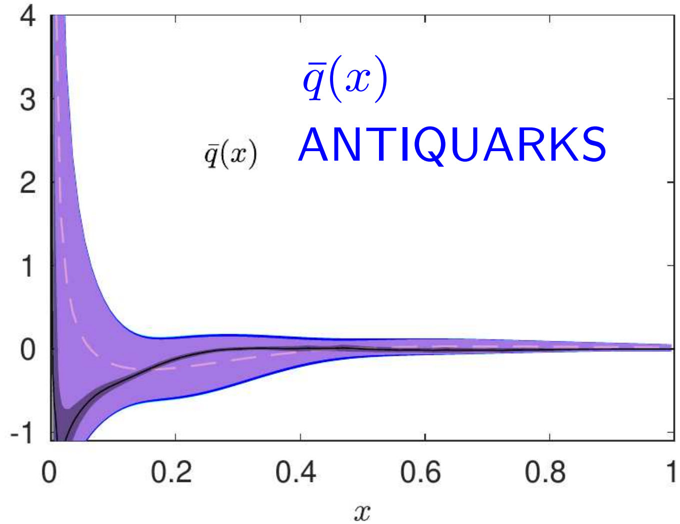
$Q^2 = 4$ GeV^2

PSEUDO



ETMC, Phys. Rev. D 103 (2021) 034510





Qualitative agreement with pheno
Systematics to be investigated

cut-off effects
truncation (matching)
higher-twist effects
reconstruction of x-dep.
finite volume effects

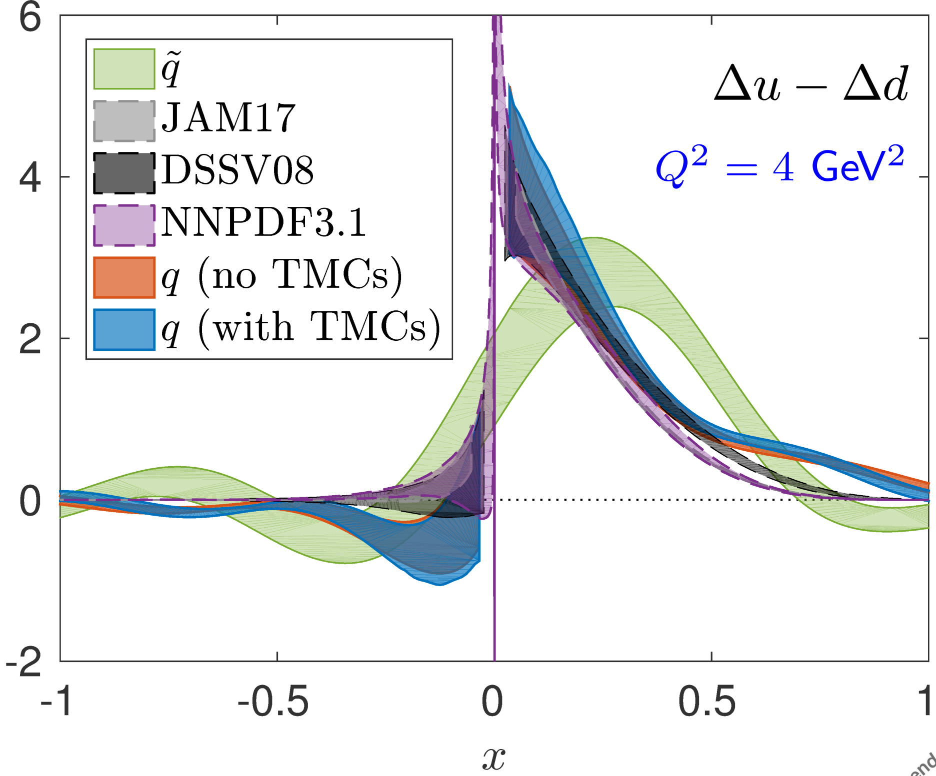
Different approach starting from the same MEs
Also: reconstruction using a pheno-inspired ansatz
And: added plausible estimates of systematics

purple – statistical error
blue – quantified systematics
cyan – estimated systematics

Quantitative agreement with phenomenology within stat. + plausible syst. error!

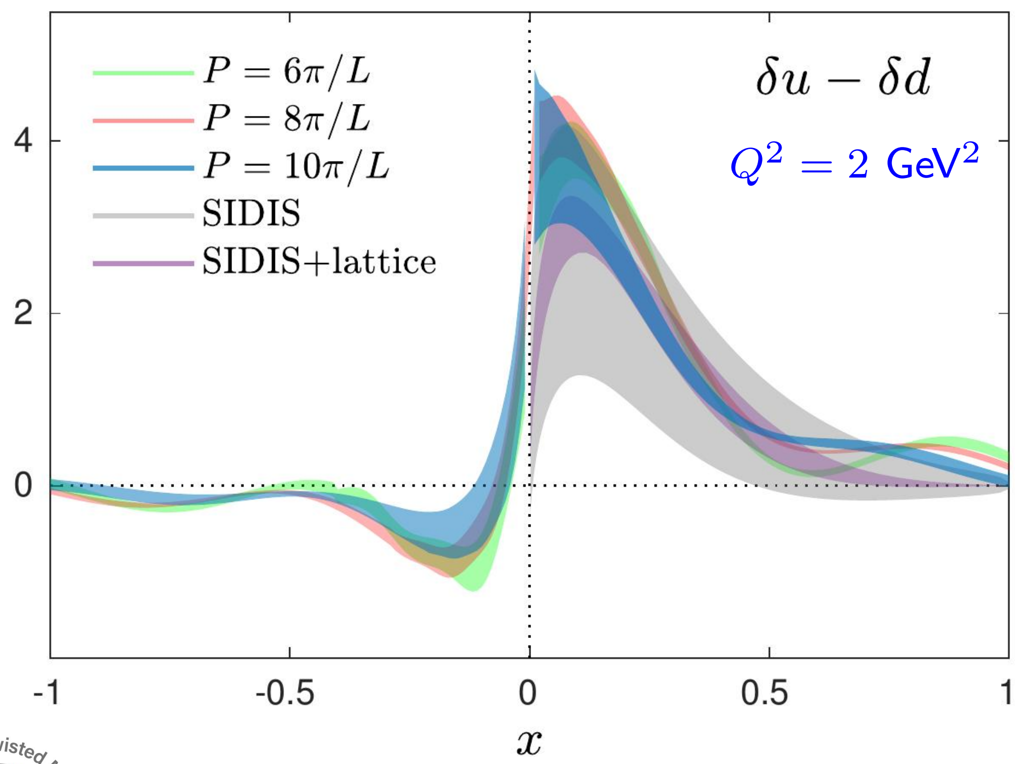
Current state-of-the-art: polarized PDFs (quasi)

Helicity PDF



ETMC, PRL121(2018)112001

Transversity PDF



ETMC, PRD98(2018)091503(R)

Same comments as for unpolarized PDFs from quasi

However, note that LQCD can play an important role for transversity PDFs

Lattice PDFs/GPDs: dynamical progress

results @ physical pion mass
results extrapolated to physical pion mass
results @ non-physical pion mass

Quasi-distributions

- Nucleon twist-2 PDFs
  - unpolarized: ETMC, PRL121(2018)112001; ETMC, PRD99(2019)114504; LP3, arXiv:1803.04393; LPC, PRD101(2020)034020; BNL/MSU, PRD102(2020)074504; ETMC, arXiv:2011.00964
  - helicity: ETMC, PRL121(2018)112001; ETMC, PRD99(2019)114504; LP3, PRL121(2018)242003; BNL/MSU, PRD102(2020)074504; ETMC, arXiv:2011.00964
  - transversity: ETMC, PRD98(2018)091503(R); ETMC, PRD99(2019)114504; LP3, arXiv:1810.05043
- Nucleon twist-3 PDFs: ETMC/Temple PRD102(2020)111501(R)
- Nucleon GPDs: ETMC PRL125(2020)262001
- Δ PDFs: ETMC/Beijing PRD102(2020)014508
- Pion/Kaon DA: LP3, PRD95(2017)094514; LP3, NPB939(2019)429
- Pion PDF: LP3, PRD100(2019)034505; BNL, PRD100(2019)034516; BNL, PRD102(2020)094513
- Pion GPD: Chen etal, NPB952(2020)114940

Pseudo-distributions

- Nucleon PDFs: JLab, PRD96(2017)094503; JLab, JHEP12(2019)081; JLab, PRL125(2020)232003; ETMC, PRD103(2021)034510
- Pion PDF: JLab, PRD100(2019)114512

Good LCSs

- Pion PDF: JLab, PRD99(2019)074507; JLab, PRD102(2020)054508

Auxiliary light quark

- Pion DA: Regensburg, EPJC78(2018)217; Regensburg, PRD98(2018)094507

Auxiliary heavy quark

- Pion DA: Taiwan/MIT, arXiv:1810.12194; Taiwan/MIT, arXiv:2009.09473

OPE without OPE

- Nucleon F_1: QCDSF, PRL118(2017)242001; QCDSF, PRD102(2020)114505

Hadronic tensor

- Nucleon W_{11}: χQCD, PRD101(2020)114503

Reviews: K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, special issue of Adv. High Energy Phys. 2019 (2019) 3036904, arXiv:1811.07248
update: M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD*, (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, arXiv:2010.02445

Some studies already advanced, but still full systematics needs to be investigated

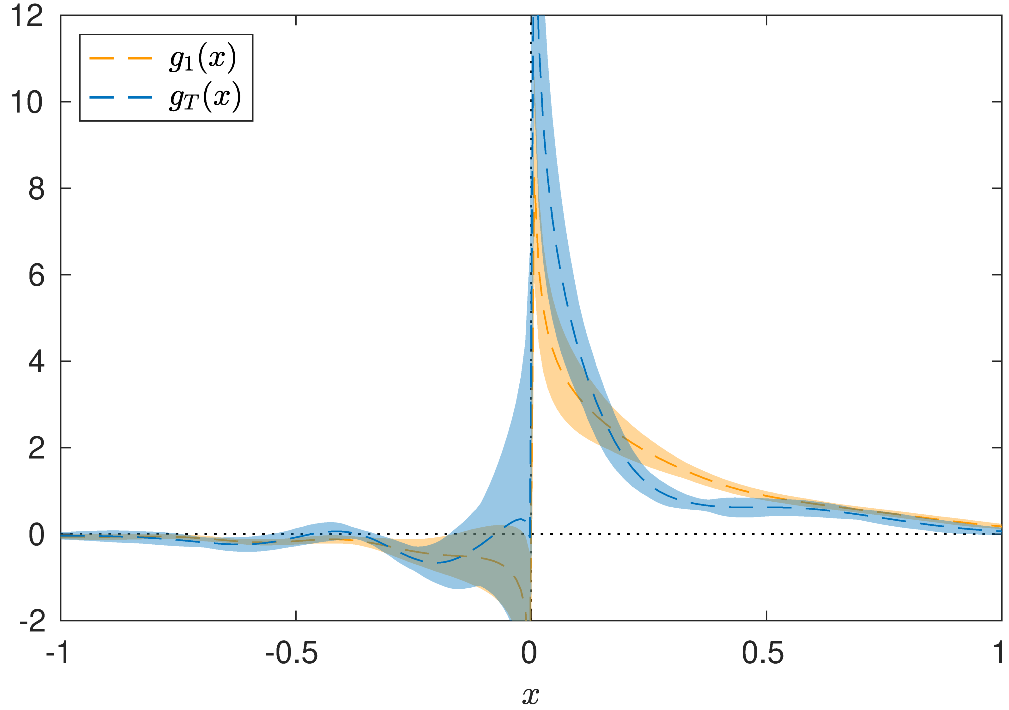
Many exploratory directions: GPDs, twist-3 PDFs/GPDs, TMDs

Exploratory direction: twist-3 PDF $g_T(x)$

Twist-2 g_1 vs. twist-3 g_T

S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz
A. Scapellato, F. Steffens

Phys. Rev. D102 (2020) 034005
Phys. Rev. D102 (2020) 111501(R)
Phys. Rev. D102 (2020) 114025



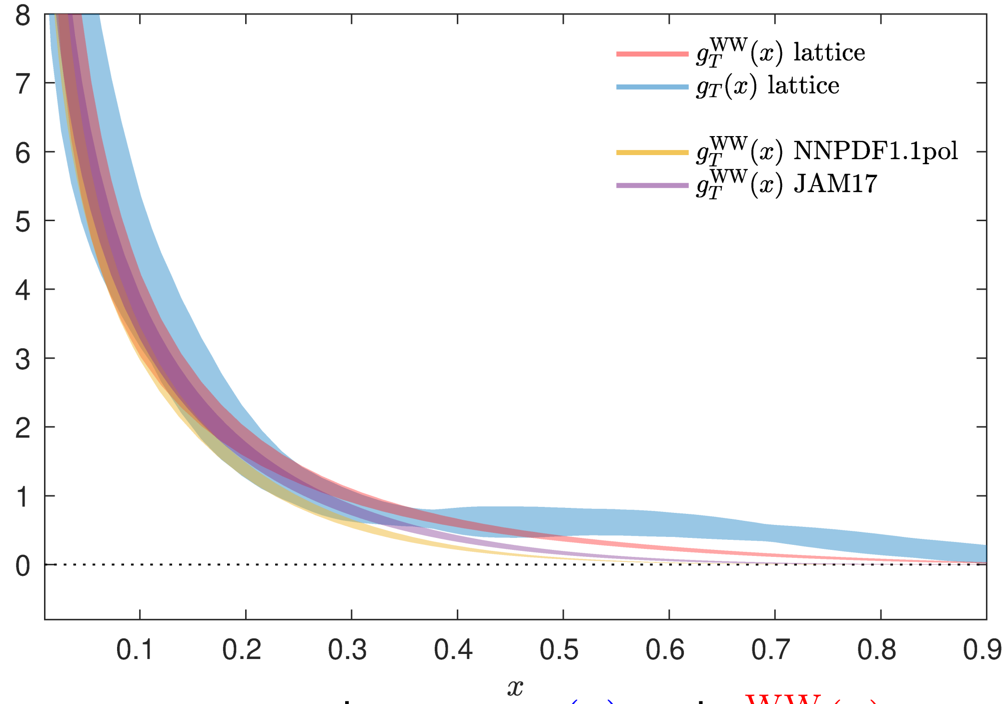
Burkhardt-Cottingham sum rule:

$$\int_{-1}^{1} dx\, g_T(x) = \int_{-1}^{1} dx\, g_1(x)$$

satisfied in our data.

Wandzura-Wilczek approx.: twist-3 $g_T(x)$ fully determined by twist-2 $g_1(x)$:

$$g_T^{\rm WW}(x) = \int_x^1 \frac{dy}{y} g_1(y)$$



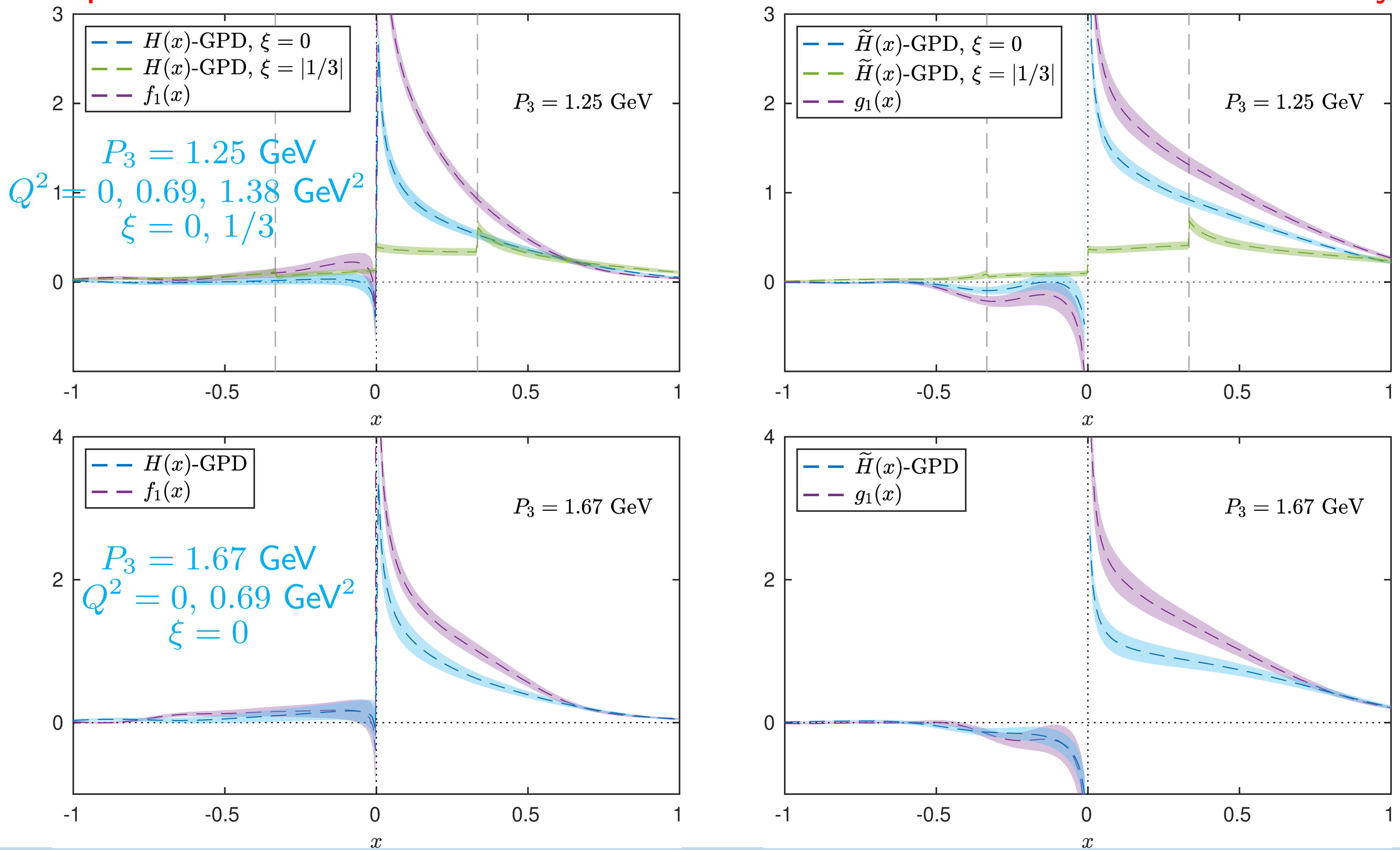
agreement between $g_T(x)$ and $g_T^{\rm WW}(x)$
for $x \lesssim 0.5$ within uncertainties
still: possible violation up to 30-40%
interestingly, similar possible violation (15-40%)
in experimental data analysis by JLab:
A. Accardi, A. Bacchetta, W. Melnitchouk, M. Schlegel, JHEP 11 (2009) 093

Exploratory direction: GPDs

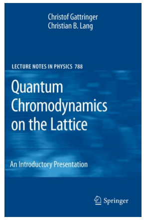
unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001

helicity



Some references



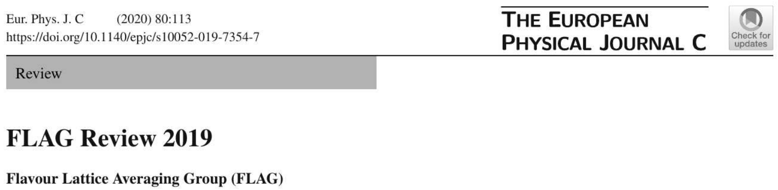
C. Gattringer, C. B. Lang
Quantum Chromodynamics on the Lattice, Springer 2009
general textbook on lattice methods

Parton distributions and lattice QCD calculations: toward 3D structure

July 8, 2020

Martha Constantinou[1,*] Aurore Courtoy[2] Markus A. Ebert[3] Michael Engelhardt[4,*] Tommaso Giani[5] Tim Hobbs[6,7] Tie-Jiun Hou[8] Aleksander Kusina[9] Krzysztof Kutak[9] Jian Liang[10] Huey-Wen Lin[11,12,*,†] Keh-Fei Liu[10] Simonetta Liuti[13] Cédric Mezrag[14] Pavel Nadolsky[6] Emanuele R. Nocera[15,*] Fred Olness[6,*] Jian-Wei Qiu[7] Marco Radici[16] Anatoly Radyushkin[7,17] Abha Rajan[18] Ted Rogers[7,17] Juan Rojo[15,19] Gerrit Schierholz[20] C.-P. Yuan[11] Jian-Hui Zhang[21] Rui Zhang[11,12] (*editors)

M. Constantinou et al.
Parton distributions and lattice QCD calculations: toward 3D structure, arXiv:2006.08636
PDFLattice20 workshop summary
– hadron structure lattice-pheno interface



Eur. Phys. J. C (2020) 80:113
https://doi.org/10.1140/epjc/s10052-019-7354-7

THE EUROPEAN PHYSICAL JOURNAL C

Review

FLAG Review 2019

Flavour Lattice Averaging Group (FLAG)

Flavor Lattice Averaging Group, FLAG19 review
Eur. Phys. J. C 80 (2020) 113, arXiv:1902.08191
broad review of lattice results

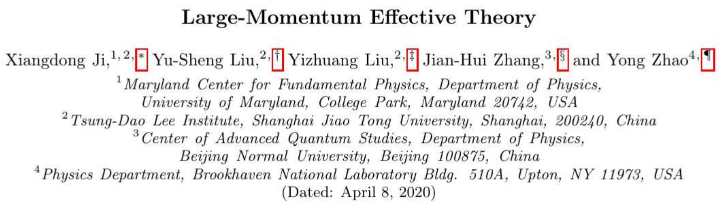
Review Article

A Guide to Light-Cone PDFs from Lattice QCD: An Overview of Approaches, Techniques, and Results

Krzysztof Cichy[1] and Martha Constantinou[2]

[1]Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland
[2]Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA

K. Cichy, M. Constantinou
A guide to light-cone PDFs from Lattice QCD
AHEP 2019 (2019) 3036904, arXiv:1811.07248
theory and status of x-dependent distributions
+ update EPJA 57(2021)77, arXiv:2010.02445



Large-Momentum Effective Theory

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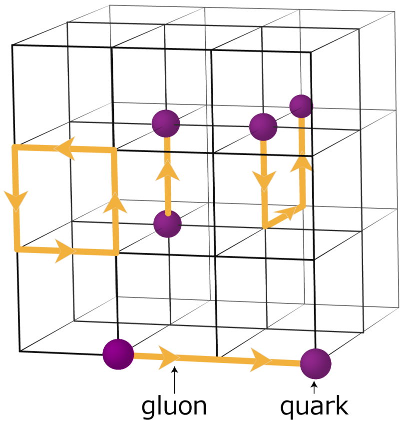
(Dated: April 8, 2020)

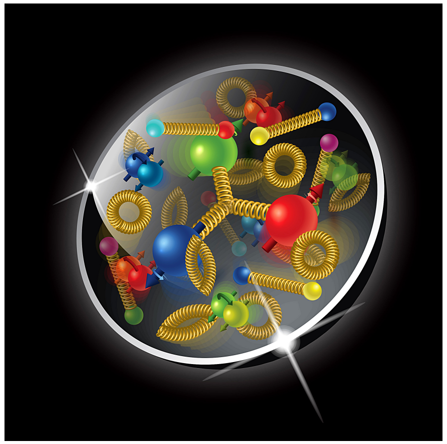
X. Ji et al.
Large-Momentum Effective Theory
arXiv:2004.03543
review of LaMET and its applications

Lattice QCD for EIC – main messages

- Lattice QCD offers a way for a careful *ab initio* study of non-perturbative aspects of QCD.
- But: pay attention to systematics!
  cut-off effects, finite volume effects, excited states, quark mass effects, isospin breaking, renormalization
- Precision vs. exploratory studies.



- Robust quantitative statements:
  low moments, form factors.
  Gives access e.g. to nucleon spin decomposition.
- x-dependence: breakthrough in recent years, but a long way to go to solid quantitative statements.
- Many exploratory directions that will eventually lead to valuable information.
- Overall, **expect complementary role of lattice.**



Thank you for your attention!